REFERENCES

1. LUR'E K.A., Optimal Control in Problems of Mathematical Physics. Moscow, Nauka, 1975.
2. LAVROV N.A., LUR'E K.A. and CHERKAEV A.V., Inhomogeneous rod of extremal torsional rigidity. Izv. Akad. Nauk SSSR, MTT, No.6, 1980.
3. PETROVA I.S. and RIKARDS R.B., Optimization of a rod with variable modulus of elasticity. Mekhanika polymerov, No.2, 1974.
4. RAMMERSTORFER F., On the optimal distribution of the Young's modulus of a vibrating, prestressed beam. J. Sound and Vibr., Vol.37, No.1, 1974.
5. NOWACKI W., Thermoelasticity. Reading, Mass., Addison-Wesley Publ. 1962.
6. MIKHLIN S.G., Variational Methods in Mathematical Physics. Moscow, Nauka, 1970.
7. MIKHLIN S.G., The Problem of the Minimum of a Quadratic Functional. Moscow-Leningrad, Gostekhizdat, 1952.
8. KUNTASHEV P.A. and NEMIROVSKII YU.V., On the solution, in terms of stresses, of the problem of the thermoelasticity of inhomogeneous bodies using the perturbation method. PMM Vol.49, No.2, 1985.
9. RUDIN W., Principles of Mathematical Analysis. N.Y., London, McGraw-Hill, 1976.
10. PSHENICHNYI B.N., Convex Analysis and Extremal Problems. Moscow, Nauka, 1980.
11. BOLEY B.A. and WEINER J.H., Theory of Thermal Stresses. N.Y. Wiley, 1960.

Translated by L.K.

PMM U.S.S.R., Vol.49, No.3, pp. 374-379, 1985
Printed in Great Britain

0021-8928/85 $10.00+0.00
Pergamon Journals Ltd.

THE PLANE PROBLEM OF ELECTROELASTICITY FOR A PIEZOELECTRIC LAYER WITH A PERIODIC SYSTEM OF ELECTRODES AT THE SURFACES*

V.A. KOKUNOV B.A. KUDRYAVTSEV AND N.A. SENIK

Static electroelasticity equations are used to study the stress state and the electric-field distribution in a piezoelectric layer at whose surface a periodic system of infinitely thin electrodes is situated. It is assumed that the layer material is piezoelectric belonging to the $6mm$ symmetry class, and the axis of symmetry is perpendicular to the middle surface of the layer. The mechanical displacements and electric potential are determined, taking the periodicity of the electrode system into account, in the form of trigonometric series, and the electrical and mechanical boundary conditions at the layer surfaces lead to the dual series equations whose solution yields the expression for the electric charge distribution density on each electrode. Formulas are given for determing the electric potential at the layer surfaces between the electrodes, and the mechanical stresses near the electrode edge. It is shown that the normal stresses at the layer surface have a singularity at the electrode edge /1/ whose presence may lead to the appearance of microcracks within this zone.

1. We shall consider the plane deformation of a piezoelectric layer $|z|<h$, $|x|<\infty$ caused by the action of the electric potential difference on the periodic system of electrodes, with the electric potentials V_0 and $-V_0$ on the upper face $z=h$ and lower face $z=-h$ of the layer (Fig.1). In the case of a piezoelectric material of class $6mm$, whose axis of symmetry coincides with the z-axis, the components of the stresses and electric induction are given by the formulas

$$\sigma_{xx}=c_{11}\frac{\partial u}{\partial x}+c_{13}\frac{\partial w}{\partial z}+e_{31}\frac{\partial\varphi}{\partial z},\quad \sigma_{zz}=c_{13}\frac{\partial u}{\partial x}+c_{33}\frac{\partial w}{\partial z}+e_{33}\frac{\partial\varphi}{\partial z} \tag{1.1}$$

$$\sigma_{zx}=c_{44}\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)+e_{15}\frac{\partial\varphi}{\partial x}$$

$$D_x=e_{15}\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)-\varepsilon_{11}{}^S\frac{\partial\varphi}{\partial x},\quad D_z=e_{31}\frac{\partial u}{\partial x}+e_{33}\frac{\partial w}{\partial z}-\varepsilon_{33}{}^S\frac{\partial\varphi}{\partial z} \tag{1.2}$$

Here $c_{11}, c_{13}, c_{33}, c_{44}$ are the moduli of elasticity, e_{31}, e_{33}, e_{15} are the piezoelectric moduli, $\varepsilon_{11}{}^S$, $\varepsilon_{33}{}^S$ are the dielectric constants, u, w are the components of the displacement vector in the direction of the x and z axes respectively, and φ is the electric potential.

The mechanical displacements u, w and electric potential are found from the system of

*Prikl.Matem.Mekhan., 49, 3, 485-491, 1985.

equations of equilibrium and of electrostatics, which can be written, taking equations (1.1) and (1.2) into account, in the form

$$c_{11}\frac{\partial^2 u}{\partial x^2}+c_{44}\frac{\partial^2 u}{\partial z^2}+(c_{13}+c_{44})\frac{\partial^2 w}{\partial x\,\partial z}+(e_{31}+e_{15})\frac{\partial^2 \varphi}{\partial x\,\partial z}=0 \tag{1.3}$$
$$(c_{13}+c_{44})\frac{\partial^2 u}{\partial x\,\partial z}+c_{44}\frac{\partial^2 w}{\partial x^2}+c_{33}\frac{\partial^2 w}{\partial z^2}+e_{33}\frac{\partial^2 \varphi}{\partial z^2}+e_{15}\frac{\partial^2 \varphi}{\partial x^2}=0$$
$$(e_{31}+e_{15})\frac{\partial^2 u}{\partial x\,\partial z}+e_{15}\frac{\partial^2 w}{\partial x^2}+e_{33}\frac{\partial^2 w}{\partial z^2}-\varepsilon_{11}{}^S\frac{\partial^2 \varphi}{\partial x^2}-\varepsilon_{33}{}^S\frac{\partial^2 \varphi}{\partial z^2}=0$$

Taking into account the symmetry of the electroelastic state relative to the plane $z = 0$ and the periodicity of the functions $u(x, z)$, $w(x, z)$, $\varphi(x, z)$ with respect to the x coordinate, we shall write the solution of system (1.3) satisfying the conditions

$$w(x, 0) = \varphi(x, 0) = \sigma_{zx}(x, 0) = 0$$

in the form of the series

$$u(x,z)=2\sum_{n=1}^{\infty}[\alpha_1 A_{1n}\operatorname{ch}(k_1\lambda_n z)+(\alpha_{21}B_{1n}-\alpha_{22}C_{1n})\operatorname{ch}(\delta\lambda_n z)\times \tag{1.4}$$
$$\cos(\omega\lambda_n z)-(\alpha_{22}B_{1n}+\alpha_{21}C_{1n})\operatorname{sh}(\delta\lambda_n z)\sin(\omega\lambda_n z)]\sin\lambda_n x$$
$$w(x,z)=W_0 z+2\Sigma_\beta,\quad \varphi(x,z)=\Phi_0 z-2\Sigma_\gamma$$
$$\Sigma_\varkappa=\sum_{n=1}^{\infty}[-\varkappa_1 A_{1n}\operatorname{sh}(k_1\lambda_n z)-(\varkappa_{21}B_{1n}-\varkappa_{22}C_{1n})\operatorname{sh}(\delta\lambda_n z)\cos(\omega\lambda_n z)+$$
$$(\varkappa_{22}B_{1n}+\varkappa_{21}C_{1n})\operatorname{ch}(\delta\lambda_n z)\sin(\omega\lambda_n z)]\cos\lambda_n x,\quad \varkappa=\beta,\gamma$$
$$\chi=\chi(k_1),\ \chi_{21}+i\chi_{22}=\chi(\delta+i\omega),\ \chi=\alpha,\beta,\gamma$$
$$\alpha(k)=a_{12}a_{23}-a_{13}a_{22},\ \beta(k)=-a_{11}a_{23}-a_{13}a_{12},\ \gamma(k)=$$
$$a_{11}a_{22}-a_{12}a_{21}$$
$$a_{11}=c_{44}k^2-c_{11},\ a_{12}=-a_{21}=k(c_{13}+c_{44}),\ a_{13}=a_{31}=$$
$$-k(e_{31}+e_{15})$$
$$a_{22}=c_{33}k^2-c_{44},\ a_{23}=-a_{32}=-e_{33}k^2+e_{15},\ a_{33}=\varepsilon_{33}{}^S k^2-\varepsilon_{11}{}^S$$

Here $\lambda_n=\pi n/L$, W_0, Φ_0, A_{1n}, B_{1n}, C_{1n} are constants and $\pm k_1$, $\pm\delta\pm i\omega$ are the roots of the equation $\det\|a_{ke}\|=0$.

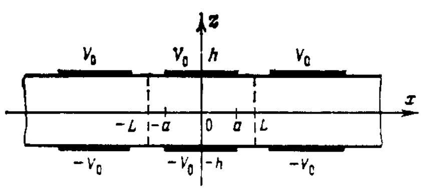

Fig.1

Using formulas (1.4) we obtain the following expressions for the stresses and the electric induction in the layer

$$\sigma_{xz}=2\Sigma_{(m)},\ \sigma_{zz}=c_{33}W_0+e_{33}\Phi_0+2\Sigma_{(m)}' \tag{1.5}$$
$$\sigma_{xx}=c_{13}W_0+e_{31}\Phi_0+2\sum_{n=1}^{\infty}\lambda_n\{-m_1k_1A_{1n}\operatorname{ch}(k_1\lambda_n z)+$$
$$[(-\delta m_2+\omega m_3)\operatorname{ch}(\delta\lambda_n z)\cos(\omega\lambda_n z)+(\delta m_3+\omega m_2)\operatorname{sh}(\delta\lambda_n z)\times$$
$$\sin(\omega\lambda_n z)]B_{1n}+[\mathrm{idem}(m_2\to -m_3,\ m_3\to m_2)]C_{1n}\}\cos\lambda_n x$$
$$D_z=e_{33}W_0-\varepsilon_{33}{}^S\Phi_0+2\Sigma_{(n)}',\quad D_x=2\Sigma_{(n)}$$
$$\Sigma_{(l)}=\sum_{n=1}^{\infty}\lambda_n\{-l_1A_{1n}\operatorname{sh}(k_1\lambda_n z)+[-l_2\operatorname{sh}(\delta\lambda_n z)\cos(\omega\lambda_n z)+$$
$$l_3\operatorname{ch}(\delta\lambda_n z)\sin(\omega\lambda_n z)]B_{1n}+[\mathrm{idem}(l_2\to -l_3,\ l_3\to l_2)]C_{1n}\}\sin\lambda_n x$$
$$\Sigma_{(l)}'=\sum_{n=1}^{\infty}\lambda_n\left\{\frac{l_1}{k_1}A_{1n}\operatorname{ch}(k_1\lambda_n z)+[(\delta' l_2+\omega' l_3)\operatorname{ch}(\delta\lambda_n z)\cos(\omega\lambda_n z)-\right.$$
$$(\delta' l_3-\omega' l_2)\operatorname{sh}(\delta\lambda_n z)\sin(\omega\lambda_n z)]B_{1n}-$$
$$[\mathrm{idem}(l_2\to l_3,\ l_3\to -l_2]C_{1n}\}\cos\lambda_n x,\quad l=m,n$$
$$\delta'=\frac{\delta}{\delta^2+\omega^2},\quad \omega'=\frac{\omega}{\delta^2+\omega^2}$$

$$m_1 = e_{15}\gamma_1 - c_{44}(\beta_1 + k_1\alpha_1),\ m_2 = e_{15}\gamma_{21} - c_{44}(\alpha_{21}\delta - \omega\alpha_{22} + \beta_{21})$$
$$m_3 = e_{15}\gamma_{22} - c_{44}(\delta\alpha_{22} + \omega\alpha_{21} + \beta_{22})$$
$$n_1 = -\varepsilon_{11}^S\gamma_1 - e_{15}(\beta_1 + k_1\alpha_1),\ n_2 = -\varepsilon_{11}^S\gamma_{21} - e_{15}(\delta\alpha_{21} - \omega\alpha_{22} + \beta_{21})$$
$$n_3 = -\varepsilon_{11}^S\gamma_{22} - e_{15}(\delta\alpha_{22} + \omega\alpha_{21} + \beta_{22})$$

Here idem $(\cdot)$ denotes the expression obtained from the expression within the preceding square brackets when the symbols are changed as shown. The following equations were used in deriving (1.5):

$$c_{11}\alpha_{21} - c_{13}(\delta\beta_{21} - \omega\beta_{22}) + e_{31}(\delta\gamma_{21} - \omega\gamma_{22}) = -\delta m_2 + \omega m_3$$
$$c_{11}\alpha_{22} - c_{13}(\delta\beta_{22} + \omega\beta_{21}) + e_{31}(\delta\gamma_{22} + \omega\gamma_{21}) = -\delta m_3 - \omega m_2$$
$$c_{11}\alpha_1 - c_{13}k_1\beta_1 + e_{31}k_1\gamma_1 = m_1 k_1$$
$$c_{13}\alpha_1 - c_{33}k_1\beta_1 + e_{33}k_1\gamma_1 = m_1/k_1$$
$$c_{13}\alpha_{21} - c_{33}(\delta\beta_{21} - \omega\beta_{22}) + e_{33}(\delta\gamma_{21} - \omega\gamma_{22}) = \delta' m_2 + \omega' m_3$$
$$c_{13}\alpha_{22} - c_{33}(\delta\beta_{22} - \omega\beta_{21}) + e_{33}(\delta\gamma_{22} + \omega\gamma_{21}) = \delta' m_3 - \omega' m_2$$
$$e_{31}\alpha_1 - e_{33}k_1\beta_1 - \varepsilon_{33}^S k_1\gamma_1 = n_1/k_1$$
$$e_{31}\alpha_{21} - e_{33}(\delta\beta_{21} - \omega\beta_{22}) - \varepsilon_{33}^S(\delta\gamma_{21} - \omega\gamma_{22}) = \delta' n_2 + \omega' n_3$$
$$e_{31}\alpha_{22} - e_{33}(\delta\beta_{22} + \omega\beta_{21}) - \varepsilon_{33}^S(\delta\gamma_{22} + \omega\gamma_{21}) = \delta' n_3 - \omega' n_2$$

Suppose there is no mechanical load at $z = \pm h$. Then the conditions

$$\sigma_{xz} = \sigma_{zz} = 0,\ z = \pm h \tag{1.6}$$

will hold, provided that we assume that

$$A_{1n} = k_1(m_2^2 + m_3^2)[\omega' \operatorname{sh}(\delta\lambda_n h)\operatorname{ch}(\delta\lambda_n h) - \delta' \sin(\omega\lambda_n h)\cos(\omega\lambda_n h)]A_n \tag{1.7}$$

$$B_{1n} = m_1\{k_1 \operatorname{sh}(k_1\lambda_n h)[(\delta' m_3 - \omega' m_2)\operatorname{ch}(\delta\lambda_n h)\cos(\omega\lambda_n h) + (\delta' m_2 - \omega' m_3)\operatorname{sh}(\delta\lambda_n h)\sin(\omega\lambda_n h)] - \operatorname{ch}(k_1\lambda_n h) \times [m_3 \operatorname{sh}(\delta\lambda_n h)\cos(\omega\lambda_n h) - m_2 \operatorname{ch}(\delta\lambda_n h)\sin(\omega\lambda_n h)]\}A_n$$

$$C_{1n} = m_1\{k_1 \operatorname{sh}(k_1\lambda_n h)[(\delta' m_2 - \omega' m_3)\operatorname{ch}(\delta\lambda_n h)\cos(\omega\lambda_n h) - (\delta' m_3 - \omega' m_2)\operatorname{sh}(\delta\lambda_n h)\sin(\omega\lambda_n h)] + \operatorname{ch}(k_1\lambda_n h)[m_3 \operatorname{ch}(\delta\lambda_n h)\sin(\omega\lambda_n h) - m_2 \operatorname{sh}(\delta\lambda_n h)\cos(\omega\lambda_n h)]\}A_n$$

$$W_0 = -e_{33}\Phi_0/c_{33}$$

Substituting expressions (1.7) into (1.5) for σ_{zz} and D_z, we obtain the electric potential and component D_z of the electric induction vector on the layer surface

$$\varphi(x, h) = \Phi_0 h - \sum_{n=1}^{\infty} f_{1n}A_n \cos\lambda_n x \tag{1.8}$$

$$D_z(x, h) = -\varepsilon_{33}^*\Phi_0 + \sum_{n=1}^{\infty}\lambda_n A_n^{(0)}\cos\lambda_n x \tag{1.9}$$

$$f_{1n} = \Phi_1 \operatorname{sh}(k_1\lambda_n h)\operatorname{sh}(2\delta\lambda_n h) + \Phi_2 \operatorname{sh}(k_1\lambda_n h)\sin(2\omega\lambda_n h) - \Phi_3 \operatorname{ch}(k_1\lambda_n h)(\operatorname{ch}(2\delta\lambda_n h) - \cos(2\omega\lambda_n h)),\ A_n^{(0)} = f_{2n}A_n$$

$$f_{2n} = d_1 \operatorname{ch}(k_1\lambda_n h)\operatorname{sh}(2\delta\lambda_n h) + d_2 \operatorname{ch}(k_1\lambda_n h)\sin(2\omega\lambda_n h) - d_3 \operatorname{sh}(k_1\lambda_n h)(\operatorname{ch}(2\delta\lambda_n h) + \cos(2\omega\lambda_n h)),\ \varepsilon_{33}^* = \varepsilon_{33}^S(1 + e_{33}^2/(c_{33}\varepsilon_{33}^S))$$

$$\Phi_1 = k_1[\gamma(m_2^2 + m_3^2)\omega' + m_1\gamma_{21}(\delta' m_3 - \omega' m_2) - m_1\gamma_{22}(\delta' m_2 + \omega' m_3)]$$
$$\Phi_2 = k_1 \text{ idem } (\omega' \to \delta',\ \delta' \to -\omega')$$
$$d_3 = m_1 k_1(\delta'^2 - \omega'^2)(m_2 n_3 - m_3 n_2)$$
$$d_1 = [n_1(m_2^2 + m_3^2)\omega' + m_1 m_2(\delta' n_3 - \omega' n_2) - m_1 m_2(\delta' n_2 + \omega' n_3)]$$

$$d_2 = \text{idem}\ (\omega' \to \delta',\ \delta' \to -\omega'),\quad \Phi_3 = m_1(\gamma_{21}m_3 - \gamma_{22}m_1)$$

We shall write the boundary conditions at the surface $z = h$ in the form

$$\varphi(x, h) = V_0,\quad 0 \leqslant x < a \tag{1.10}$$

$$D_z(x, h) = 0,\quad a < x < L \tag{1.11}$$

Then, taking into account (1.8), (1.9) we can conclude that the conditions (1.10), (1.11) lead to dual series equations for determining the coefficients $A_n^{(0)}$

$$\sum_{n=1}^{\infty} F_n A_n^{(0)} \cos\lambda_n x = V_0 - \Phi_0 h,\quad 0 \leqslant x < a;\quad F_n = \frac{f_{1n}}{f_{2n}} \tag{1.12}$$

$$-\varepsilon_{33}{}^*\Phi_0 + \sum_{n=1}^{\infty} \lambda_n A_n^{(0)} \cos\lambda_n x = 0,\quad a < x < L \tag{1.13}$$

2. Passing now to the problem of solving the dual equations (1.12), (1.13), we write them in the form

$$\sum_{n=1}^{\infty} A_n^{(0)} \cos\lambda_n x = \beta_*(V_0 - \Phi_0 h) + \sum_{n=1}^{\infty} R_n A_n^{(0)} \cos\lambda_n x,\quad 0 \leqslant x < a \tag{2.1}$$

$$-\varepsilon_{33}{}^*\Phi_0 + \sum_{n=1}^{\infty} \lambda_n A_n^{(0)} \cos\lambda_n x = 0,\quad a < x < L \tag{2.2}$$

$$R_n = 1 - \beta_* F_n,\quad \lim_{n\to\infty} R_n = 0,\quad \beta_* = \frac{d_1 - d_3}{\Phi_1 - \Phi_3}$$

We introduce the auxilliary function $f(x)$, assuming that

$$-\varepsilon_{33}{}^*\Phi_0 + \sum_{n=1}^{\infty} \lambda_n A_n^{(0)} \cos\lambda_n x = f(x),\quad 0 \leqslant x < a \tag{2.3}$$

Then from (2.2) and (2.3) we obtain

$$-\varepsilon_{33}{}^*\Phi_0 = \frac{1}{L}\int_0^a f(\xi)\, d\xi,\quad \lambda_n A_n^{(0)} = \frac{2}{L}\int_0^a f(t)\cos\lambda_n t\, dt \tag{2.4}$$

Substituting (2.4) into (2.1) and using the expression /2/

$$\sum_{n=1}^{\infty} \frac{\cos\lambda_n x \cos\lambda_n t}{\lambda_n} = -\frac{L}{2\pi}\ln(2\,|\cos 2x_* - \cos 2t_*|) \tag{2.5}$$

$$(x_* = \pi x/(2L),\ t_* = \pi t/(2L))$$

we obtain the integral equation in $f(x)$

$$-\frac{1}{\pi}\int_0^a f(t)\ln(2\,|\cos 2x_* - \cos 2t_*|)\, dt = \beta_*(V_0 - \Phi_0 h) + \frac{2}{L}\sum_{n=1}^{\infty} \frac{R_n \cos\lambda_n x}{\lambda_n}\int_0^a f(t)\cos\lambda_n t\, dt,\quad 0 \leqslant x < a \tag{2.6}$$

To solve Eq.(2.6) we introduce new variables ξ and ζ, connected with x and t by the following relations:

$$\cos 2x_* = \cos^2 a_* + \sin^2 a_* \cos 2\xi_* \tag{2.7}$$

$$\cos 2t_* = \cos^2 a_* - \sin^2 a_* \cos 2\zeta_*$$

$$\left(\xi_* = \frac{\pi\xi}{2L},\ \zeta_* = \frac{\pi\zeta}{2L},\ a_* = \frac{\pi a}{2L}\right)$$

Changing to the variables ξ, ζ, we shall use the expansion

$$-\frac{L}{2\pi}\ln(2\,|\cos 2x_* - \cos 2t_*|) = -\frac{L}{2\pi}\ln(2\sin^2 a_*) - \sum_{k=1}^{\infty} \frac{\cos\lambda_k\xi \cos\lambda_k\zeta}{\lambda_k}$$

and write Eq.(2.6) in the form

$$-\frac{1}{\pi}\ln(2\sin^2 a_*)\int_0^L f^*(\zeta)\, d\zeta - \frac{2}{\pi}\int_0^L f^*(\zeta)\left(\sum_{k=1}^{\infty} \frac{\cos\lambda_k\xi\cos\lambda_k\zeta}{k}\right) d\zeta = \tag{2.8}$$

$$\beta_*(V_0 - \Phi_0 h) + \sum_{n=1}^{\infty} \frac{R_n}{\lambda_n}\cos(\lambda_n x(\xi))\,\frac{2}{L}\int_0^L f^*(\zeta)\cos(\lambda_n t(\zeta))\, d\zeta$$

$$(f^*(\zeta) = f(t(\zeta))\, t'(\zeta))$$

We shall seek the solution of (2.8) in the form of a series

$$f(t(\zeta))\,t'(\zeta) = \sum_{m=0}^{\infty} \alpha_m \cos\lambda_m\zeta \qquad (2.9)$$

$$\alpha_0 = \frac{1}{L}\int_0^a f(t)\,dt = -\varepsilon_{33}{}^*\Phi_0$$

Then, taking into account the expansions /3/

$$\cos(\lambda_n x(\xi)) = \sum_{k=0}^{\infty} \beta_k^{(n)}\cos\lambda_k\xi,\quad \cos(\lambda_n t(\zeta)) = \sum_{s=0}^{\infty}\beta_s^{(n)}\cos\lambda_s\zeta \qquad (2.10)$$

$$(n = 1, 2, \ldots,\ \beta_k{}^{(n)} = 0 \quad \text{for} \quad k > n)$$

we obtain, from (2.8), an infinite system of algebraic equations for determining the constants α_m

$$\alpha_0\left[\ln(2\sin^2 a_*) - \frac{\pi h\beta_*}{L\varepsilon_{33}{}^*} + 2\sum_{n=1}^{\infty}\frac{R_n}{n}(\beta_0^{(n)})^2\right] + \sum_{s=1}^{\infty}\alpha_s\sum_{n=1}^{\infty}\frac{R_n}{n}\beta_0^{(n)}\beta_s^{(n)} = -\frac{\pi\beta_*}{L}V_0 \qquad (2.11)$$

$$2\alpha_0\sum_{n=1}^{\infty}\frac{R_n}{n}\beta_m^{(n)}\beta_0^{(n)} - \frac{\alpha_m}{m} + \sum_{s=1}^{\infty}\alpha_s\sum_{n=1}^{\infty}\frac{R_n}{n}\beta_m^{(n)}\beta_s^{(n)} = 0 \qquad (2.12)$$

$$(m = 1, 2, \ldots)$$

Returning in (2.9) to the variable t and using the well-known relations for the Chebyshev polynomials T_{2m} [2], we obtain the following expression for the electric charge density at the upper electrode system:

$$f(t) = \frac{\cos t_*}{\sqrt{\cos^2 t_* - \cos^2 a_*}}\sum_{m=0}^{\infty}(-1)^m\alpha_m T_{2m}\left(\frac{\sin t_*}{\sin a_*}\right) \qquad (2.13)$$

Taking into account (2.13), we obtain the solution of the system (2.1), (2.2) in the form

$$A_n^{(0)} = [2\alpha_0\beta_0^{(n)} + \sum_{s=1}^{n}\alpha_s\beta_s^{(n)}]\,\lambda_n^{-1} \qquad (2.14)$$

Let us now determine the electric and normal stress σ_{xx} at the layer surface $z = h$. Using (1.5), (1.7), (1.8) we find

$$\varphi(x, h) = \Phi_0 h - \frac{1}{\pi\beta_*}\int_0^a f(t)\ln(2\,|\cos 2x_* - \cos 2t_*|)\,dt - \beta_*^{-1}\sum_{n=1}^{\infty}R_n A_n^{(0)}\cos\lambda_n x \qquad (2.15)$$

$$\sigma_{xx}(x, h) = \sigma_{xx}^{(0)} - \frac{m_1(m_2{}^2 - m_3{}^2)}{d_1 - d_3}(2k_1\omega\delta' - \omega - k_1{}^2\omega')\,\eta(a-x)f(x) - \frac{m_1(m_2{}^2 + m_3{}^2)}{d_1 - d_3}\sum_{n=1}^{\infty}G_n A_n^{(0)}\lambda_n\cos\lambda_n x \qquad (2.16)$$

$$\sigma_{xx}^{(0)} = \left[e_{31}{}^* + \frac{m_1(m_2{}^2 + m_3{}^2)}{d_1 - d_3}(2k_1\omega\delta' - \omega - k_1{}^2\omega')\,\varepsilon_{33}{}^*\right]\Phi_0$$

$$f_{2n}G_n = g_1\,[\mathrm{ch}\,(k_1\lambda_n h)\,\mathrm{sh}\,(2\delta\lambda_n h) - \mathrm{sh}\,(k_1\lambda_n h)\,(\mathrm{ch}\,(2\delta\lambda_n h) + \cos(2\omega\lambda_n h))] + g_2\mathrm{ch}\,(k_1\lambda_n h)\sin(2\omega\lambda_n h)$$

$$g_1 = 2k_1\omega\delta' d_1 - (\omega + k_1{}^2\omega')d_3,\quad e_{31}{}^* = e_{31}(1 - c_{13}e_{33}/(c_{33}e_{31}))$$

$$g_2 = (2k_1\omega\delta' - \omega - k_1{}^2\omega')d_2 - (\delta - k_1{}^2\delta')(d_1 - d_3)$$

($\eta(x)$ is a unique function). Eq.(2.16) implies that the stress $\sigma_{xx}(x, h)$ has a singularity at the edge of the electrode.

3. A numerical analysis of the electroelastic fields in the strip was carried out for the piezoelectric material PZT-4 [4] for $a/h = 3$, $L/h = 18$. In the series representing the coefficients accompanying the unknowns $\alpha_0, \alpha_1, \ldots$ of system (2.11), (2.12), only the first four were retained. The truncated system was used to determine $\alpha_0, \ldots, \alpha_3$, and then

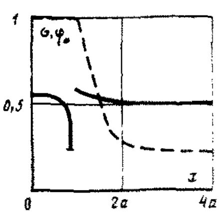

Fig.2

$$\varphi(x, h) = \Phi_0 h - \frac{\alpha_0 L}{\pi\beta_*}\ln(2\sin^2 a_*) + \beta_*^{-1}\sum_{n=1}^{\infty}\left\{\frac{\alpha_n}{\lambda_n}(-1)^n T_{2n}(\gamma_*) - R_n A_n^{(0)}\cos\lambda_n x\right\}$$

$$0 \leqslant x < a,\quad (\gamma_* = \sin x_*/\sin a_*)$$

was used to confirm condition (1.10). The discrepancy in satisfying condition (1.10) did not exceed 1% for all values $0 \leqslant x < a$. In the region outside the electrodes, the electric field potential at $z = h$ was calculated using formula

$$\varphi(x, h) = \Phi_0 h - \beta_*^{-1} \sum_{n=1}^{\infty} R_n A_n^{(0)} \cos \lambda_n x - \tag{3.1}$$
$$- \beta_*^{-1} \sum_{m=0}^{\infty} (-1)^m \alpha_m \int_0^{\pi/2} \cos(2m\eta) \ln(4 \,|\cos^2 \eta \sin^2 a_* - \sin^2 x\,|)\, d\eta$$

obtained from (2.15), (2.13) after transforming the integral term. We note that the quadratures appearing in (3.1) can be computed for $m = 0, 1$ ($|x| > a$) Fig.2 shows how $\varphi_* = \varphi/V_0$ changes with x when $z = h$ (dashed line). The solid lines show the variation in the stress $\sigma = \sigma_{xx} h/(V_0 e_{31})$, computed from (2.16). Analysis of the numerical results shows that at the edge of the electrode the stress σ_{xx} has a root-type singularity caused by the change in the electrical boundary conditions.

REFERENCES

1. KUDRYAVTSEV B.A., Electroelastic state of a half-plane, made of piezoceramic material, with two boundary electrodes. Problemy prochnosti, No.7, 1982.
2. GRADSHTEIN I.S. and RYZHIK I.M., Tables of Integrals, Sums, Series and Products, Moscow, Fizmatgiz, 1963.
3. HUSSAIN M.A. and PU S.L., Dynamic stress intensity factor for an unbounded plate having collinear cracks. Engng Fract. Mech., Vol.4, No.4, 1972.
4. BERLINKUR D., CURRAN D. and JAFFE G., Piezoelectric and piezomagnetic materials and their use in transducers. In: Physical Acoustics. Vol.1, Ch. A.M., Mir, 1966.

Translated by L.K.